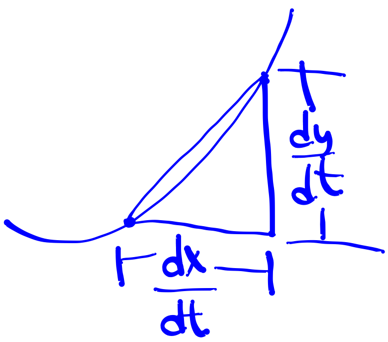
7.4) Arc Length of curves defined by parametric equations



Arc length =

$$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

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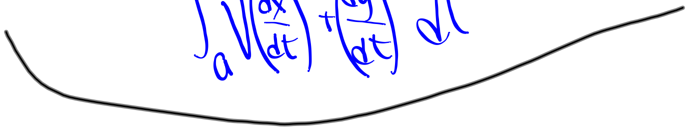


$$x(t) = 3\cos(t) \quad y(t) = 3\sin(t)$$

$$0 \leq t \leq 2\pi$$

$$\frac{dx}{dt} = -3\sin(t) \quad \frac{dy}{dt} = 3\cos(t)$$

$$A_L = \int_0^{2\pi} \sqrt{(-3\sin t)^2 + (3\cos t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9(\sin^2 t + \cos^2 t)}\, dt$$

$$= \int_0^{2\pi} 3\, dt = 3t\Big|_0^{2\pi} = 6\pi$$

$$[2\pi r = C]$$

cardiod $x = 2\cos(t) - \cos(2t)$

$y = 2\sin(t) - \sin(2t)$

deltoid $x = 2\cos(t) + \cos(2t)$

$y = 2\sin(t) - \sin(2t)$

$((\sin(t))^2 + 1)$ $x = \dfrac{\sqrt{2}\cos t}{\sin^2 t + 1}$ $y = \dfrac{\sqrt{2}\cos t \sin t}{\sin^2 t + 1}$

lemniscate

$x = 2\sin(1.5t + 2)$

$y = 3\sin(t)$

$0 \le t \le 8\pi$

astroid $x = \cos^3(t)$ $y = \sin^3(t)$

7.4/7 $24xy = y^4 + 48 \quad y \in [2,4]$

Imp Diff. wrt x

$$24y + 24x\frac{dy}{dx} = 4y^3\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{24y}{4y^3 - 24\left(\frac{y^4+48}{24y}\right)}$$

$$= \frac{24y^2}{4y^4 - (y^4+48)} = \frac{8y^2}{y^4-16}$$

7.4/7 $24xy = y^4 + 48 \quad y \in [2,4]$

find $\frac{dx}{dy}$

$\frac{dy}{dy} = 1$

$$24\frac{dx}{dy}\cdot y + 24x = 4y^3$$

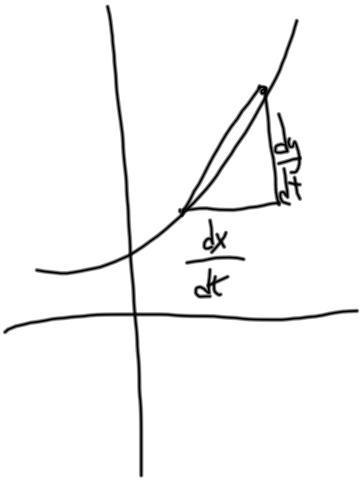
$$\frac{dx}{dy} = \frac{4y^3 - 24x}{24y}$$

$$= \frac{4y^3 - 24\left(\frac{y^4+48}{24y}\right)}{24y}$$

$$= \frac{4y^4 - (y^4+48)}{24y^2} = \frac{3y^4 - 48}{24y^2} = \frac{y^4-16}{8y^2}$$

$$\therefore AL = \int_2^4 \sqrt{1 + \left(\frac{y^4-16}{8y^2}\right)^2}\, dy$$

7.4 Arc Length of Curves defined parametrically



$$\text{Arc Length} = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

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$$AL = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Example:

$x = 3\cos(t) \quad y = 3\sin(t)$

$\underset{a}{0} \leq t \leq \underset{b}{2\pi}$

Graph & find arclength

$x = 3\cos(t) \qquad y = 3\sin(t)$

$\frac{dx}{dt} = -3\sin(t) \qquad \frac{dy}{dt} = 3\cos(t)$

$$AL = \int_a^b \sqrt{(-3\sin(t))^2 + (3\cos(t))^2}\, dt$$

$$\int_a^b \sqrt{9\sin(t)^2 + 9\cos(t)^2}\, dt$$

$$\int_a^b \sqrt{9(\sin(t)^2 + \cos(t)^2)}\, dt$$

$$\int_a^b \sqrt{9 \cdot 1}\, dt$$

$$\int_a^b 3\, dt = 3x\Big|_a^b$$

wilder's fault

$(3 \cdot 2\pi) - (3 \cdot 0) = 6\pi$